International Journal of Theoretical Physics, Vol. 36, No. 6, 1997

$ISO(3, 1/N)$ Supergravity Lagrangian with Noether Coupling

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Received September 22, 1996

By making use of the Noether coupling method and introducing the interaction of gauge field and fermionic field, we formulate the $ISO(3, 1/N)$ supergravity Lagrangian and verify its symmetry.

1. GAUGE GROUP AND GAUGE FIELDS

The gauge group of the supergravity considered here is $ISO(3, 1/N)$, which is constructed from two subgroups [space-time Poincaré group $ISO(3, 1)$ and inner-symmetry group $SO(N)$] and supersymmetry transformation.

According to the supergravity (Shao, 1981, 1990), let the $ISO(3, 1/N)$ group generators and respective gauge field and gauge field strengths be

$$\tau_{AB} = (M_{ab}, P_a, E_i, H_{i\alpha})$$

$$B_\mu^{AB} = (B_\mu^{ab}, V_\mu^a, E_\mu^i, \Lambda_\mu^{i\alpha})$$

$$R_{\mu\nu}^{AB} = (R_{\mu\nu}^{ab}(\mu), R_{\mu\nu}^a(p), R_{\mu\nu}^i(E), R_{\mu\nu}^{i\alpha}(H))$$

where B_μ^{ab}, V_μ^a are Poincaré gauge fields, $\Lambda_\mu^{i\alpha}$ are gravitino fields, and E_μ^i are Yang–Mills fields. Then we have

$$B_\mu = B_\mu^{AB}\tau_{AB} = \tfrac{1}{2}B_\mu^{ab}M_{ab} + V_\mu^a P_a + E_\mu^i E_i + k\Lambda_\mu^{i\alpha} H_{i\alpha} \tag{1}$$

$$\begin{aligned} R_{\mu\nu} &= R_{\mu\nu}^{AB}\tau_{AB} = R_{\mu\nu}^{ab}(M)M_{ab} + R_{\mu\nu}^a(P)P_a + R_{\mu\nu}^i(E)E_i + KR_{\mu\nu}^{i\alpha}(H)H_{i\alpha} \\ &= D_\mu B_\nu - D_\nu B_\mu \end{aligned} \tag{2}$$

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0020-7748/97/0600-1401$12.50/0

where K is a coupling constant whose dimension is -1, and $D_\mu = \partial_\mu - ig^{(AB)}B_\mu^{AB}\tau_{AB}$ is the $ISO(3, 1/N)$ covariant derivative. By making use of the commutator of τ_{AB} (Shao, 1981), and comparing to the coefficient of (2), we have the gauge field strength components

$$
\begin{aligned}
R_{\mu\nu}^{ab}(M) &= \partial_\mu B_\nu^{ab} + B_{\mu c}^{a} B_\nu^{cb} - \mu \leftrightarrow \nu \\
R_{\mu\nu}^{a}(P) &= \partial_\mu V_\nu^a + B_{\nu b}^{a} V_\mu^b - \mu \leftrightarrow \nu - \tfrac{1}{2}(\overline{\Lambda}_{\mu i}\gamma^a\Lambda_{\nu i} - \mu \leftrightarrow \nu) \qquad (3) \\
R_{\mu\nu}^{i}(E) &= \partial_\mu E_\nu^i - \partial_\nu E_\mu^i + f_{jk}^{i} E_\mu^j E_\nu^k \\
R_{\mu\nu}^{i\alpha}(H) &= \overline{\Lambda}^{i\alpha}\overleftarrow{D}'_\mu + E_\mu^k(g_k)_j^i \Lambda_\nu^{j\alpha} - \mu \leftrightarrow \nu
\end{aligned}
$$

where $\overleftarrow{D}'_\mu = \overleftarrow{\partial}_\mu - B_\mu^{ab}(\sigma_{ab})^T$.

By introducing the local parameters corresponding to the $ISO(3, 1/N)$ generators

$$\epsilon^{AB} = (L^{ab}, b^a, I^i, \pi^{i\alpha})$$

we have the transformation law of gauge field and gauge field strength:

$$
\begin{aligned}
B_\mu \to B'_\mu &= UB_\mu U^{-1} + iU\partial_\mu U^{-1} \\
R_{\mu\nu} \to R'_{\mu\nu} &= UR_{\mu\nu}U^{-1}
\end{aligned}
$$

where $U = \exp(-i\epsilon^{AB}\tau_{AB})$ represents $ISO(3, 1/N)$ group elements. Then the infinitesimal transformations are

$$
\begin{aligned}
\delta B_\mu &= -i[\epsilon^{AB}\tau_{AB}, B_\mu\} - \partial_\mu\epsilon^{AB}\tau_{AB} \\
\delta R_{\mu\nu} &= -i[\epsilon^{AB}\tau_{AB}, R_{\mu\nu}\} \qquad (4)
\end{aligned}
$$

2. SUPERSYMMETRY AND INNER-SYMMETRY TRANSFORMATION

Let $L^{ab} = 0$, $b^a = 0$, $I^i = 0$, $\pi^{i\alpha} \neq 0$; the transformation is a pure supersymmetry transformation. Then from (4), we find the transformation laws of each gauge field under supersymmetry:

$$\delta_S B_\mu^{ab} = 0, \qquad \delta_S V_\mu^a = i\pi^{i\alpha}\Lambda_{i\mu}^{\beta}(\gamma_c^a)_{\alpha\beta}$$

$$\delta_S E_\mu^i = 0, \qquad \delta_S \Lambda_\mu^{i\alpha} = i\pi^{i\alpha}\overleftarrow{D}'_\mu \qquad (5)$$

and those of the field strengths:

$$\delta_S R_{\mu\nu}^{ab}(M) = 0, \qquad \delta_S R_{\mu\nu}^{a}(P) = i\pi^{i\alpha}R_{\mu\nu i}^{\beta}(H)(\gamma_c^a)_{\alpha\beta} \qquad (6)$$

When $L^{ab} = 0$, $b^a = 0$, $I^i \neq 0$, $\pi^{i\alpha} = 0$, in the same way, we have the pure inner-symmetry transformation laws

$$\begin{aligned} \delta_I B^{ab}_\mu &= 0 & \delta_I E^i_\mu &= -\partial_\mu I^i - if^i_{jk} I^j E^k_\mu \\ \delta_I V^a_\mu &= 0 & \delta_I \Lambda^{i\alpha}_\mu &= ikI^j (g_j)^i_k \Lambda^{k\alpha}_\mu \\ \delta_I R^{ab}_{\mu\nu}(M) &= 0 & \delta_I R^i_{\mu\nu}(E) &= -if^i_{jk} I^j R^k_{\mu\nu}(E) \\ \delta_I R^a_{\mu\nu}(P) &= 0 & \delta_I R^{i\alpha}_{\mu\nu}(H) &= ikI^j (g_i)^i_k R^{k\alpha}_{\mu\nu}(H) \end{aligned} \tag{7}$$

3. THE SYMMETRY OF THE GAUGE FIELD AND MATTER FIELD LAGRANGIAN

In the supersymmetry theory the physical system includes the fermionic coordinates $\psi^{i\alpha}$ in superspace time, which can describe the particle field (Salam and Stra, 1975). They are anticommutative Majorana spinors

$$\{\psi^{i\alpha}, \psi^{i\beta}\} = 0, \qquad \psi^{i\alpha^c} = \psi^{i\alpha}$$

and their supersymmetry and inner-symmetry transformations are (Shao, 1981)

$$\delta_S \psi^{i\alpha} = R_{\mu\nu} \sigma^{\mu\nu} \pi^{i\alpha} \tag{8}$$

$$\delta_I \psi^{i\alpha} = -iI^i E_j \psi^{i\alpha} \tag{9}$$

Then we can define the gauge and matter field Lagrangian

$$\mathscr{L}_0 = -\tfrac{1}{4} R^{AB}_{\mu\nu} R^{\mu\nu}_{AB} + \tfrac{1}{2} \bar{\psi}_{i\alpha} \partial\!\!\!/ \psi^{i\alpha} \tag{10}$$

where $\partial\!\!\!/ = \gamma^\mu \partial_\mu$ and

$$R^{AB}_{\mu\nu} R^{\mu\nu}_{AB} = R^{ab}_{\mu\nu}(M) R^{\mu\nu}_{ab}(M) + R^i_{\mu\nu}(E) R^{\mu\nu}_i(E) + R^{i\alpha}_{\mu\nu}(H) R^{\mu\nu}_{i\alpha}(H)$$

and $R^a_{\mu\nu}(P) = 0$ in the nontorsion space. Obviously it is an invariant under space-time and Lorentz transformation.

Now we verify that the Lagrangian is an invariant under the supersymmetry and inner-symmetry transformations, respectively.

3.1. Supersymmetry Transformation

Taking the supersymmetry transformation of $\mathscr{L}_0$,

$$\begin{aligned} \delta_S \mathscr{L}_0 &= \delta_S(-\tfrac{1}{4} R^{AB}_{\mu\nu} R^{\mu\nu}_{AB} + \tfrac{1}{2} \bar{\psi}_{i\alpha} \partial\!\!\!/ \psi^{i\alpha}) \\ &= -\delta_S[\partial_\mu B^{AB}_\mu R^{\mu\nu}_{AB}] - \tfrac{1}{2} \bar{\pi}_{i\alpha} R_{\nu\lambda} \sigma^{\nu\lambda} \gamma^\mu \partial_\mu \psi^{i\alpha} + \tfrac{1}{2} \bar{\psi}_{i\alpha} \gamma^\mu \partial_\mu R_{\nu\lambda} \sigma^{\nu\lambda} \pi^{i\alpha} \end{aligned} \tag{11}$$

and making use of the following formulas, the Bianchi identity, and the Euler equation,

$$\begin{aligned} \sigma^{\nu\lambda}\gamma^{\mu} &= \tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}\gamma_{\rho}\gamma_5 - \tfrac{1}{2}(\eta^{\nu\mu}\gamma^{\lambda} - \eta^{\lambda\mu}\gamma^{\nu}) \\ \gamma^{\mu}\sigma^{\nu\lambda} &= \tfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}\gamma_{\rho}\gamma_5 - \tfrac{1}{2}(\eta^{\mu\lambda}\gamma^{\nu} - \eta^{\mu\nu}\gamma^{\lambda}) \\ \overline{\psi}^{i\alpha}\gamma_{\mu}\gamma_5\pi_{i\alpha} &= \overline{\pi}_{i\alpha}\gamma_{\mu}\gamma_5\psi^{i\alpha} \\ \overline{\psi}^{i\alpha}\gamma_{\mu}\pi_{i\alpha} &= -\overline{\pi}_{i\alpha}\gamma_{\mu}\psi^{i\alpha} \\ \epsilon^{\mu\nu\lambda\rho}\partial_{\mu}R_{\nu\lambda} &= 0 \\ \partial_{\mu}R^{\mu\nu} &= 0 \end{aligned} \tag{12}$$

we have

$$\mathcal{L}_0 = -i\pi^{i\alpha}\partial_{\mu}(D'_{\nu}R^{\mu\nu}_{i\alpha}(H)) = \pi^{i\alpha}\partial_{\mu}K^{\mu}_{i\alpha}$$

where $K^{\mu}_{i\alpha} = -iD'_{\nu}R^{\mu\nu}_{i\alpha}(H)$ is the superconservation current. Therefore the action of $\mathcal{L}_0$ is an invariant of the supersymmetry transformation, that is,

$$\delta_S S_0 = \int \delta_S\mathcal{L}_0 \, d^4X = \int \pi^{i\alpha}\, \partial_{\mu}K^{\mu}_{i\alpha}\, d^4X = 0 \tag{13}$$

3.2. Inner-Symmetry Transformation

Taking the inner-symmetry transformation for (10)

$$\delta_I\mathcal{L}_0 = -\tfrac{1}{4}\delta_I(R^{AB}_{\mu\nu}R^{\mu\nu}_{AB}) + \tfrac{1}{2}\delta_I(\overline{\psi}_{i\alpha}\, \partial\!\!\!/\psi^{i\alpha}) \tag{14}$$

and substituting (9) into (14), we find that its second term is zero; then

$$\begin{aligned} \delta_I\mathcal{L}_0 &= -\tfrac{1}{4}\delta_I(R^{AB}_{\mu\nu}R^{\mu\nu}_{AB}) \\ &= \partial_{\mu}[I^i(\partial_{\nu}R^{\mu\nu}_i(E) + if^k_{ij}E^j_{\nu}R^{\mu\nu}_k(E) - iK(g_i)^k_j\Lambda^{j\alpha}_{\nu}R^{\mu\nu}_k(H))] \\ &= I^i\partial_{\mu}K^{\mu}_i \end{aligned}$$

where

$$K^{\mu}_i = \partial_{\nu}R^{\mu\nu}_i(E) + if^k_{ij}E^i_{\nu}R^{\mu\nu}_k(E) - iK(g_i)^k_j\Lambda^{j\alpha}_{\nu}R^{\mu\nu}_k(H) \tag{15}$$

is the Yang–Mills conservative current. Therefore $\mathcal{L}_0$ is invariant under the $ISO(3, 1/N)$ gauge group transform.

4. SUPERGRAVITY LAGRANGIAN

In accordance with the supergravity theory (Freedman *et al.*, 1976; Deser and Zumino, 1976) we choose the ERS local supergravity Lagrangian

$$\mathcal{L}_{sg} = -\frac{1}{4K} eR - \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \overline{\pi}^{i\alpha}_{\mu} \gamma_5 \gamma_\nu D'_\lambda \Lambda_{\rho i \alpha} = \mathcal{L}_{(1)} + \mathcal{L}_{(2)} \tag{16}$$

where $D'_\lambda = \partial_\lambda - B^{ab}_{\mu}(\sigma_{ab})^T$.

Now we verify that the action of $\mathcal{L}_{sg}$ is an invariant under the expressions (5) and (6):

$$\delta_S \mathcal{L}_{(1)} = \delta_S \left(\frac{1}{16K} \epsilon^{\mu\nu\lambda\rho} \epsilon_{abcd} V^{\alpha}_{\mu} V^{b}_{\nu} R^{cd}_{\lambda\rho}(M) \right)$$

$$= \frac{i}{8} \epsilon^{\mu\nu\lambda\rho} \epsilon_{abcd} \overline{\pi}^{i\alpha} \Lambda^{i\alpha}_{\mu} (\gamma^a c)_{\alpha\beta} V^b_\nu R^{cd}_{\lambda\rho}(M) \tag{17}$$

$$\delta_S \mathcal{L}_{(2)} = -\frac{1}{4} \epsilon^{\mu\nu\lambda\rho} [\delta_S \overline{\Lambda}^{i\alpha} \gamma_5 \gamma_\nu R^{i\alpha}_{\lambda\rho}(H) + \overline{\Lambda}^{i\alpha}_{\mu} \gamma_5 \gamma_b R^{i\alpha}_{\lambda\rho}(H) \delta_S V^b_\nu$$
$$+ \overline{\Lambda}^{i\alpha}_{\mu} \gamma_5 \gamma_\nu \delta_S R^{i\alpha}_{\lambda\rho}(H)] \tag{18}$$

Substituting (5) into (18), we have for the first term

$$-\frac{i}{4} \epsilon^{\mu\nu\lambda\rho} \overline{\pi}^{i\alpha} [\overleftarrow{\partial}_\mu - B^{ab}_{\mu}(\sigma_{ab})^T \gamma_5 \gamma_\nu R^{i\alpha}_{\lambda\rho}(H)]$$

$$= -\frac{i}{4} \epsilon^{\mu\nu\lambda\rho} [\partial_\mu \overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu R^{i\alpha}_{\lambda\rho}(H) - \overline{\pi}^{i\alpha} \gamma_5 \gamma_b R^{i\alpha}_{\lambda\rho}(H) \partial_\mu V^b_\nu$$
$$- \overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu \partial_\mu R^{i\alpha}_{\lambda\rho}(H) - \overline{\pi}^{i\alpha} B^{ab}_{\mu} (\sigma_{ab})^T \gamma_5 \gamma^\nu R^{i\alpha}_{\lambda\rho}(H)]$$

$$= -\frac{i}{4} \epsilon^{\mu\nu\lambda\rho} [\partial_\mu (\overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu R^{i\alpha}_{\lambda\rho}(H)) - \overline{\pi}^{i\alpha} \gamma_5 \gamma_b R^{i\alpha}_{\lambda\rho}(H) \partial_\mu V^b_\nu$$
$$- \overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu D'_\mu R^{i\alpha}_{\lambda\rho}(H) - \overline{\pi}^{i\alpha} B^{ab}_{\mu} \gamma_5 [\sigma_{ab}, \gamma_\nu] R^{i\alpha}_{\lambda\rho}(H)]$$

$$= -\frac{i}{4} \epsilon^{\mu\nu\lambda\rho} \partial_\mu (\overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu R^{i\alpha}_{\lambda\rho}(H) - \frac{i}{4} \epsilon^{\mu\nu\lambda\rho} \overline{\Lambda}^{i\alpha} \gamma_5 \Lambda^{\beta}_{i\gamma} \overline{\pi}^{i\alpha} \gamma^a \gamma_5 R^{i\alpha}_{\lambda\rho}(H))$$
$$+ \frac{1}{16} \epsilon^{\mu\nu\lambda\rho} \overline{\pi}^{i\alpha} \gamma_5 \gamma_\nu R^{ab}_{\lambda\rho} (\sigma_{ab})^T \Lambda^{i\alpha}_{\mu} \tag{19}$$

where we made use of the nontorsion condition (11), and

$$[\sigma_{ab}, \gamma_c] = -i2\eta_{ab}\gamma_c + i2\eta_{bc}\gamma_a$$

For the second term, using the Fierz transposition formulas and the symmetry of the Majorana spinor, we have

$$-\frac{i}{4}\epsilon^{\mu\nu\lambda\rho}\overline{\Lambda}_{\mu}^{i\alpha}\gamma_5\gamma_b R_{\lambda\rho}^{i\alpha}(H)\delta_S V_{\nu}^{b}$$

$$= \frac{i}{4}\epsilon^{\mu\nu\lambda\rho}\overline{\Lambda}_{\mu}^{i\alpha}\gamma_5\Lambda_{i\nu}^{\beta}\pi^{i\alpha}\gamma^{a}\gamma_5 R_{\lambda\rho}^{i\alpha}(H) \tag{20}$$

For the third term, substituting (5), we have

$$\frac{i}{16}\epsilon^{\mu\nu\lambda\rho}\overline{\pi}^{i\alpha}\gamma_5\sigma_{ab}\gamma_{\nu}R_{\lambda\rho}^{ab}(M)\Lambda_{\mu}^{i\alpha} \tag{21}$$

and (18) becomes

$$\delta_S\mathscr{L}_{(2)} = -\frac{1}{4}\epsilon^{\mu\nu\lambda\rho}\partial_{\mu}(\overline{\pi}^{i\alpha}\gamma_5\gamma_{\nu}R_{\lambda\rho}^{i\alpha}(H))$$

$$-\frac{i}{8}\epsilon^{\mu\nu\lambda\rho}\epsilon_{abcd}\overline{\pi}^{i\alpha}\gamma^{d}\Lambda_{\mu}^{i\alpha}V_{\nu}^{c}R_{\lambda\rho}^{ab}(M)$$

so the supersymmetry variation of (16) is

$$\delta_S\mathscr{L}_{sg} = \delta_S\mathscr{L}_{(1)} + \delta_S\mathscr{L}_{(2)} = -\frac{i}{4}\epsilon^{\mu\nu\lambda\rho}\partial_{\mu}(\overline{\pi}^{i\alpha}\gamma_5\gamma_{\nu}R_{\lambda\rho}^{i\alpha}(H))$$

Then the action of $\mathscr{L}_{sg}$ is an invariant of supersymmetry:

$$\delta_S S_{sg} = \int \delta_S\mathscr{L}_{sg}\, d^4X = 0$$

5. NOETHER COUPLING CURRENT AND *ISO*(3, 1/*N*) LAGRANGIAN

It is well known that the supersymmetry Noether current is

$$J_{i\alpha}^{\mu} = \frac{\delta\mathscr{L}_0}{\delta\partial_{\mu}\psi^{i\alpha}}\Delta\psi + \frac{\delta\mathscr{L}_0}{\delta\partial_{\mu}\Lambda_{\nu}^{i\alpha}}\Delta\Lambda_{\nu} - K_{i\alpha}^{\mu} \tag{22}$$

where $\Delta\psi$ and $\Delta\Lambda_{\nu}$ are relative to the supersymmetry transform

$$\delta_S\psi^{i\alpha} = \pi^{i\alpha}\Delta\psi, \qquad \delta_S\Lambda_{\nu}^{i\alpha} = \pi^{i\alpha}\Delta\Lambda_{\nu}$$

Then $\Delta\psi = R_{\mu\nu}\sigma^{\mu\nu}$ and $\Delta\Lambda_{\nu} = i(\partial_{\nu} - B_{\nu}^{ab}(\sigma_{ab})^{T})$. Substituting into (22), we have

$$J_{i\alpha}^{\mu} = \tfrac{1}{2}\overline{\psi}_{i\alpha}\gamma^{\mu}R_{\nu\lambda}\sigma^{\nu\lambda} \tag{23}$$

Coupling the gravitino field with the fermion, we obtain

$$\mathcal{L}_{NS} = K\Lambda_{\mu}^{i\alpha} J_{i\alpha}^{\mu} = \tfrac{1}{2} K \Lambda_{\mu}^{i\alpha} \bar{\psi}_{i\alpha} \gamma^{\mu} R_{\nu\lambda} \sigma^{\nu\lambda}$$

The Yang–Mills Noether current is

$$J_i^{\mu} = \frac{\delta \mathcal{L}_0}{\delta \partial_{\mu} \psi_{\nu}^{i\alpha}} \Delta\psi^{\alpha} + \frac{\delta \mathcal{L}_0}{\delta \partial_{\mu} E_{\nu}^{i}} \Delta E - K_i^{\mu}$$

Substituting (7) and (9) into (24), we find

$$J_i^{\mu} = -\frac{i}{2} \bar{\psi}_{i\alpha} E_j \psi^{j\alpha} + iK(g_i)_j^k \Lambda_{\nu}^{i\alpha} R_{k\alpha}^{\mu\nu}(H) \tag{24}$$

so the Yang–Mills Lagrangian is

$$\begin{aligned} \mathcal{L}_{NI} &= KE_{\mu}^{i} J_i^{\mu} \\ &= -\frac{i}{2} KE_{\mu}^{i} \bar{\psi}_{i\alpha} E_j \psi^{j\alpha} + iK^2 E_{\mu}^{i} (g_i)_j^k \Lambda_{\nu}^{j\alpha} R_{k\alpha}^{\mu\nu}(H) \end{aligned}$$

Then we finally obtain the local gauge-transform-invariant *ISO*(3, 1/*N*) Lagrangian

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{sg} + \mathcal{L}_{NS} + \mathcal{L}_{NI} \tag{25}$$

where

$$\mathcal{L}_0 = -\frac{1}{4} R_{\mu\nu}^{AB} R_{AB}^{\mu\nu} + \frac{1}{2} \bar{\psi}_{i\alpha} \not{\partial} \psi^{i\alpha}$$

$$\mathcal{L}_{sg} = -\frac{1}{4K} eR - \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} \bar{\Lambda}_{\mu}^{i\alpha} \gamma_5 \gamma_{\nu} D'_{\lambda} \Lambda_{\rho i\alpha}$$

$$\mathcal{L}_{NS} = \frac{1}{2} K \Lambda_{\mu}^{i\alpha} \bar{\psi}_{i\alpha} \gamma^{\mu} R_{\nu\lambda} \sigma^{\nu\lambda}$$

$$\mathcal{L}_{NI} = -\frac{i}{2} KE_{\mu}^{i} \bar{\psi} E_j \psi^{j\alpha} + iK^2 E_{\mu}^{i} (g_i)_j^k \Lambda_{\nu}^{j\alpha} R_{k\alpha}^{\mu\nu}(H)$$

Furthermore, we find that the supersymmetry and inner-symmetry charges of this theory are

$$H_{i\alpha} = \int d^3X\, J_{i\alpha}^{0} = \int d^3X \frac{1}{2} \bar{\psi}_{i\alpha} \gamma^0 R_{\nu\lambda} \sigma^{\nu\lambda}$$

$$E_i = \int d^3X\, J_i^{0} = \int d^3X \left(\frac{i}{2} \bar{\psi}_{i\alpha} E_j \psi^{j\alpha} + iK(g_i)_j^k \Lambda_{\nu}^{i\alpha} R_{k\alpha}^{0\nu}(H) \right)$$

6. CONCLUSION

The expression (25) leads to more interaction between the fields, particularly that of the fermionic field with the others. The new Noether Lagrangians $\mathscr{L}_{NS}$ and $\mathscr{L}_{NI}$ introduce interaction between the ferimonic field and gauge fields. $\mathscr{L}_{NS}$ mainly introduces fermionic and gravitino fields, including their three-vertex and four-vertex interactions. $\mathscr{L}_{NI}$ mainly introduces the three-vertex interaction between the fermionic and Yang–Mills field, and introduces three-vertex and four-vertex interactions between gravitino and Yang–Mills fields, which is different from other gravitational theories. Because there is a GR Einstein term in $\mathscr{L}_{sg}$, this Lagrangian can include those four kinds of interactions.

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